

# Gravitating lepton bag model

Alexander Burinskii,

Theor. Phys. Lab., NSI, Russian Academy of Sciences,  
B. Tulkaya 52, Moscow 115191 Russia, e-mail: burinskii@mail.ru

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## Abstract

As is known, the gravitational and electromagnetic (EM) field of the Dirac electron is described by an over-extremal Kerr-Newman (KN) black hole (BH) solution which has the naked singular ring and two-sheeted topology. This space is regulated by the formation of a regular source based on the Higgs mechanism of broken symmetry. This source shares much in common with the known MIT- and SLAC-bag models, but has the important advantage, of being in accordance with gravitational and electromagnetic field of the external KN solution. The KN bag model is flexible. At rotations, it takes the shape of a thin disk, and similar to other bag models, under deformations it creates a string-like structure which is positioned along the sharp border of the disk.

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## 1 Introduction and Overview

It has been discussed for long time that black holes (BH) are to be related with elementary particles [1]. The Kerr-Newman (KN) rotating BH solution paid in this respect especial interest, since, as it was shown by Carter [2], its gyromagnetic ratio  $g = 2$  corresponds to the Dirac electron, and therefore, the four measurable parameters of the electron: spin  $J$ , mass  $m$ , charge  $e$  and magnetic moment  $\mu$  indicate that gravitational and electromagnetic field of the electron should be described by the KN solution. In the recent paper [3]

Dokuchaev and Eroshenko considered a solution of the Dirac equation under BH horizon, and suggested that this model may represent a “...particle-like charged solutions in general relativity...”. On the other hand, it should be noted that the model of a Dirac particle confined under BH horizon can also be considered as a type of gravitating bag model, and it acquires especial interest since this bag is to be gravitating, leading to a progress beyond the known MIT and SLAC bag models [4, 5]. However, the spin and charge of elementary particles are very high with respect to their masses, which prevents formation of the BH horizons. In particular, the Kerr-Newman solution with parameters of electron: charge  $e$ , mass  $m$ , and spin parameter  $a = J/m$  exceeds the threshold value  $e^2 + a^2 \leq m^2$  for existence of the horizons about 21 order. Similar ratios for the other elementary particles show that besides the Higgs boson, which has neither spin nor charge, none of the elementary particles may be associated with a true black hole, and rather, they should be associated with the over-rotating Kerr geometry, with  $|a| \gg m$ .

The corresponding over-rotating KN space has topological defect – the naked Kerr singular ring, which forms a branch line of space into two sheets described by different metrics: the sheet of advanced and sheet of the retarded fields. The Kerr singular and related two-sheeted structure created the problem of a mysterious source of the Kerr and KN solutions, which paid considerable attention during more than four decades [6, 7, 8, 9, 10, 11, 12, 13, 14]. For the story of this investigation, see for example [15]. Long-term attempts to resolve the puzzle of the source of Kerr geometry led first to the model of the vacuum bubble – the rotating disk-like shell [9, 8]. In the subsequent works, the vacuum state inside the bubble turned into a superconducting bulk formed of a condensate of the Higgs field, [13, 14]. The structure of source acquired typical features of the soliton and Q-ball models, becoming similar to the known bag models [4, 5].

Recent analysis of the Dirac equation inside the KN soliton source [16], confirmed that the regularized KN solution shares much in common with the known MIT and SLAC bag models. However, the *gravitating bag* formed by the KN bubble-source should have specific features associated with the *need to preserve the external KN field*.

On the other hand, the semiclassical theory of the bag models [5] includes elements of quantum theory which are *based on a flat space-time* without gravity, and we are faced with a known conflict between gravity and quantum theory. Our solution to this problem in [13, 14] is based on two requirements:

**I:** The space-time should be flat inside the bag,

**II:** The space-time outside the bag should be the exact KN solution.

Therefore the quantum-gravity conflict is resolved by separation of their regions of influence. Remarkably, *these requirements determine features of the KN bag unambiguously*. First of all, they determine uniquely border of the KN bag, showing explicitly that, in accord with general concept of the bag models [5, 17], the KN bag has to be flexible and its shape depends on the rotation parameter  $a = J/m$ , as well as from the local intensity of the electromagnetic (EM) field.

As a result, for parameters of an electron, the rotating bag takes the shape of a thin disk of ellipsoidal form, see Fig.1. Its thickness  $R$  turns out to be equal to classical radius of electron  $r_e = e^2/2m$ , while radius of the disk corresponds to the Compton wave-length of the disk,<sup>1</sup> which allows to identify it with a *dressed electron*.

The the degree of oblateness of this disk is  $a/R = \alpha^{-1} = 137$ , the fine structure constant  $\alpha$  acquires a geometric interpretation.

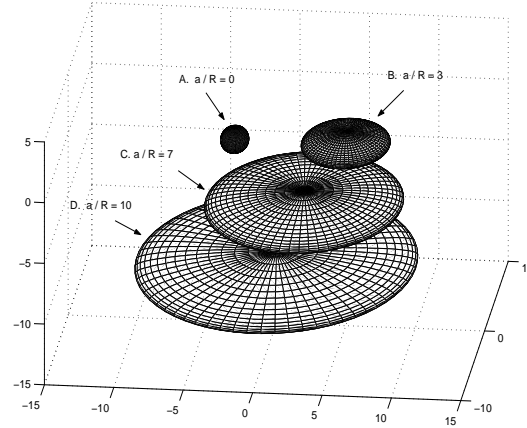


Figure 1: (A): Spherical bad with zero rotation,  $a/R = 0$ , and the rotating disk-like bags for different ratios  $a/R$ , (B):  $a/R = 3$ ; (C):  $a/R = 7$ , (D):  $a/R = 10$ .

The next very important consequence of these requirements is the emergence of a ring-string structure on the bag border, [18], and further the

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<sup>1</sup>This was determined by Lòpez [9].

emergence of a singular pole associated with traveling-wave excitations of the string [44]. This pole can be associated with a single quark, and finally, the KN bag takes the form of a coherent “bag-string-quark” system.

Finally, these requirements determine that the Higgs condensate should be enclosed *inside the bag*, contrary to the standard treatments of the bag as *a cavity in the Higgs condensate*, [4]. Realization of this requirement cannot be done with the usual quartic potential for self-interaction of the Higgs field [4, 5], and requires a more complicate field model, based on a few chiral fields and a supersymmetric scheme of phase transition [19].

At this point we have to mention the important role of Kerr Theorem, which determines the null vector field  $k_\mu(x)$ , the Kerr Principal congruence which forms a vortex polarization of Kerr-Schild (KS) metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu. \quad (1)$$

The Kerr theorem gives two solutions for this congruence  $k_\mu^\pm$ , which determine two sheets of the KN solution corresponding to two different metrics  $g_{\mu\nu}^\pm$ . Solutions of the Dirac equation on the KN background should be consistent with the metric corresponding to one of this congruence.

We show that two solutions of the Kerr theorem generate two massless Weyl spinor fields which are coupled into a Dirac field consistent with the Kerr geometry. However, the null spinor fields of the Kerr congruences are massless, and there appears the question on the origin of the mass term. Answer comes from theory of the bag models [5]), where the Dirac mass is a variable depending on the local vev of the Higgs condensate.

This gives a direct hint to a consistent embedding of the Dirac equation in the regularized KN background, indicating that the both sheets of the KN solution are necessary as *carriers of the initially massless leptons*. It turns out in agreement with the basic concepts of the Glashow-Weinberg-Salam model [20], in which the lepton masses are generated by the Higgs mechanism of symmetry breaking.

As a result, we conclude that two-sheeted Kerr’s structure is an essential element for the consistent with gravity space-time realization of the electroweak sector of the Standard Model.

## 2 Over-rotating Kerr geometry: two-sheeted structure and regular source

The KN solution in the Kerr-Schild (KS) form [21] has the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \quad (2)$$

where  $\eta_{\mu\nu}$  is metric of auxiliary Minkowski space,  $x^\mu = (t.x.y.z) \in M^4$ ,<sup>2</sup> and

$$H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta}. \quad (3)$$

The vector field  $k_\mu$  is null,  $k_\mu k^\mu = 0$ , and determined by the differential form

$$k = k_\mu dx^\mu = dr - dt - a \sin^2 \theta d\phi, \quad (4)$$

where  $t, r, \theta, \phi$ , are the Kerr oblate spheroidal coordinates

$$x + iy = (r + ia)e^{i\phi} \sin \theta, \quad z = r \cos \theta, \quad t = \rho - r. \quad (5)$$

The field  $k^\mu(x)$  forms Principal Null Congruence (PNC) [22],  $\mathcal{K}$ , which determines polarization of the Kerr space-time, see Fig.2. The PNC is focussed at the Kerr singular ring,  $r = 0$ ,  $\cos \theta = 0$ , which is the branch line of the Kerr space into two sheets  $r > 0$  and  $r < 0$ .<sup>3</sup>

Extension of the Kerr congruence to negative sheet of the KS space ( $r < 0$ ) along the lines  $\phi = \text{const.}$ ,  $\theta = \text{const.}$  creates another congruence with different radial direction, and the congruence which is *outgoing* by  $r > 0$  turns on the negative sheet into *ingoing* one.<sup>4</sup> Thus, the Kerr solution describes in the KS form two different sheets of space-time, determined by two different congruences

$$k_\mu^\pm(x) dx^\mu = \pm dr - dt - a \sin^2 \theta d\phi \quad (6)$$

and two different metrics

$$g_{\mu\nu}^\pm = \eta_{\mu\nu} + 2Hk_\mu^\pm k_\nu^\pm \quad (7)$$

on the same Minkowski background  $x^\mu \in M^4$ .

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<sup>2</sup>We use signature  $(-+++)$ .

<sup>3</sup>These are Riemannian sheets of the Kerr complex radial distance  $\tilde{r} = r + ia \cos \theta$ .

<sup>4</sup>The relations (5) are changed too, [22].

This two-sheetedness created the problem of the source of Kerr geometry, and there appeared two lines of investigation. One of them, [10, 11, 23], accepted two-sheetedness as indication of its plausible connection with a spinor structure of the Kerr space-time and with two-sheeted structure of the topologically nontrivial “Alice” strings introduced by Schwarz and Witten [24].

Alternative line of the investigation was related with truncation of the KN negative sheet, and with a consistent replacement of the excised region by a source in agreement with the Einstein-Maxwell field equations, [6, 7, 8, 9, 12, 13, 14].

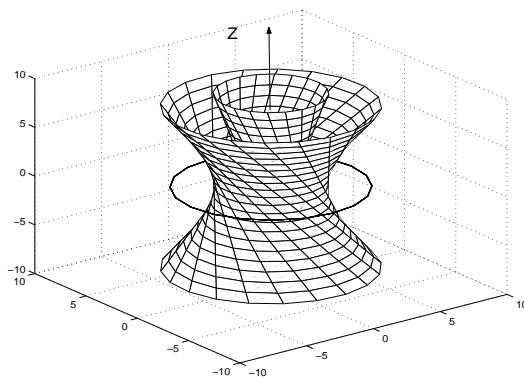


Figure 2: Kerr’s principal congruence of the null lines (twistors) is focused on the Kerr singular ring, forming a branch line of the Kerr space into two sheets.

There is a freedom in choice of the truncating surface, and in the most successful version of the model suggested by López [9], the KN source formed a bubble, boundary of which was determined by matching the external KN metric (2) with a flat metric inside the bubble. According to (2) and (3), this boundary has to be placed at the radius  $r = R = \frac{e^2}{2m}$ .

From (5), one sees that  $r$  is indeed the oblate spheroidal coordinate determined by the equation

$$\frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} = 1, \quad (8)$$

and the source of KN solution takes the form of a very oblate disk of radius  $r_c \approx a = \frac{1}{2m}$  with thickness

$$r_e = e^2/2m, \quad (9)$$

which is the classical radius of electron. So, that the fine structure constant acquires geometrical meaning as the degree of oblateness of the disk-like source,  $r_e/r_c = e^2 \approx 137^{-1}$ .

As a result of the regularization, the disk-like region surrounding the Kerr singular ring is excised and replaced by flat space, which acts as a cut-off parameter – an effective minimal distance  $R = r_e$  to the former Kerr singular ring. Note that for the case without rotation,  $a = 0$ , the disk-like bubble takes the spherical form and size of classical electron (9).

The López model was later on transformed into a *soliton-bubble* model [13, 14], in which the thin shell of the bubble was replaced by a field model of a domain wall providing a smooth phase transition between the external KN solution and the flat internal space. This phase transition was modelled by Higgs mechanism of broken symmetry, and flat interior of the KN bubble was formed by a supersymmetric state of the Higgs condensate.

The field model of broken symmetry is similar to Landau-Ginzburg model of superconductivity [25], and regularization of the singular KN solution can be viewed as an analogue to the Meissner effect, expulsion of the gravitational and EM fields from interior of the superconducting source.

### 3 Higgs condensate and mass of the Dirac field

The used for regularization of the KN solution Higgs mechanism of broken symmetry relates the source of KN solution with many other extended particle-like models of the electroweak sector of the standard model. In particular, with the superconducting string model of Nielsen and Olesen [25, 26], with Coleman’s Q-ball models, [28, 27, 29, 30, 31] and with the famous MIT- and SLAC- bag models. In this paper we pay especial attention to fermionic sector of the KN source and obtain close similarity between the Higgs mechanism of mass generation in the KN soliton model and that in the SLAC bag model [5].

Hamiltonian of the SLAC model for interaction of the Higgs field with the Dirac field  $\psi$  has the form

$$\mathcal{H} = \int d^3x \{ \psi^\dagger (-i\vec{\alpha} \cdot \vec{\nabla} + g\beta\sigma) \psi + \frac{1}{2}(\dot{\sigma}^2 + |\vec{\nabla}\sigma|^2) + V(\sigma) \}, \quad (10)$$

where  $g$  is a dimensionless coupling parameter, and self-interaction of the

non-linear Higgs field  $\Phi$  is described by quartic potential

$$V(|\Phi|) = g(\bar{\sigma}\sigma - f^2)^2, \quad (11)$$

where  $\sigma = \langle |\Phi| \rangle$  is vacuum expectation value (vev) of the Higgs field. The true vacuum state of the Higgs field  $\sigma = 0$  is not the state of lowest energy, and the Higgs field is triggered in the state  $\sigma = f$ , which breaks the gauge symmetry of the spinor field  $\psi$ . As a result, fermion acquires the mass  $m = g\eta$  which is used in the confinement mechanism of the bag models. However, the condensate of the Higgs field  $\sigma = f$  breaks also the gauge symmetry of the EM fields. In the known bag models, it turns the external EM fields in the short-range one which distorts the external KN solution.

For example, in the MIT-bag model the Higgs vev vanishes inside the bag,  $r < R$ , and takes nonvanishing value  $\sigma = f$ , in *outer region*,  $r > R$ . See. Fig.3.

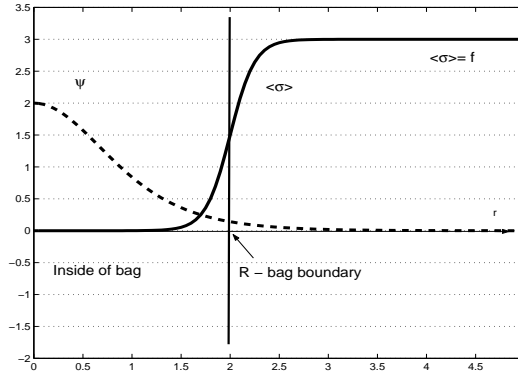


Figure 3: Positions of the vev of Higgs field  $\sigma$  and the confined spinor wave function  $\Psi$  (quark) in the MIT-bag model.

The Dirac equation in the presence of the  $\sigma$ -field takes the form

$$(i\gamma^\mu \partial_\mu - g\sigma)\psi = 0, \quad (12)$$

and the Dirac wave function  $\psi$  turns out to be massless inside the bag and acquires a large effective mass  $m = gf$  outside. The quarks are confined inside the bag, where they have the most energetically favorable position.

Geometry of the Higgs vacuum state is different in the SLAC bag models, see Fig.4. The vev  $\sigma$  gives the mass to Dirac field outside the bag as well as



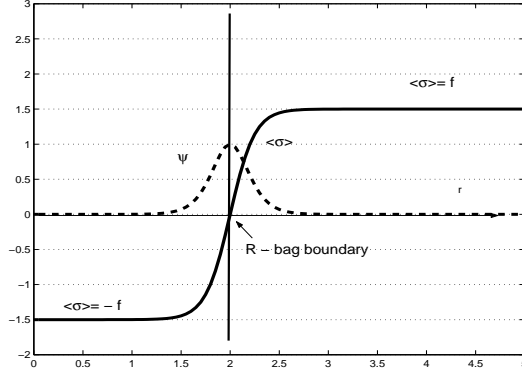


Figure 4: Classical solutions of the SLAC-bag model. The vacuum field  $\sigma$  and the localized spinor (quark) wave function confined to the thin shell - boundary of the bag.

inside. The mass vanishes only in the very narrow region near the surface of the bag,  $r \sim R$ .

Such geometry of the broken vacuum state creates a sharp localization of the Dirac wave function at the border of the bag.

In the bag models we are faced with several very important novelties:

(A) The statement on impossibility of localization of the Dirac wave function beyond the distances comparable with the Compton wave length  $\hbar/mc$  is violated, and quarks can localize within a very thin region at the bag shell. The reason of that is scalar nature of the confinement potential, for which “...there is no Klein paradox of the familiar type encountered in the presence of strong, sharp vector potential.” [5].

(B) There is effectively used a semi-classical approach to one-particle Dirac theory. Solving Dirac equation for a quark in scalar potential assumes that all the negative-energy states are filled, and treatment is focused on the lowest positive-energy eigenvalues. Therefore, “...there is no ambiguity in identifying and interpreting the desired positive energy ”one-particle” solutions.” [5, 32]

(C) Mass term of the Dirac equation (12) is determined by the vev of Higgs field  $\sigma(x) = \langle |\Phi(x)| \rangle$ , and therefore, it turns out to be a function in configuration space.

(D) Bag models presumed to be very soft, compliable and extensible. They are easily deformed, and under rotations and deformations they may

acquire extended stringy structures accompanied by vibrations [17].

All these peculiarities of the bag models are compatible with the soliton-bubble source of KN solution. However, there is one important difference: the typical bag model represents a bubble or cavity in a superconducting media – Higgs condensate, while in the gravitating bubble-source of the KN solution the Higgs condensate is enclosed *within the bubble*, leaving unbroken the true vacuum outside the bag.

In the MIT and SLAC bag models, the Higgs condensate is placed outside the source, and the external vacuum represents a superconducting state, which leads to the short-range external EM fields.

A dual (turned inside out) geometry was suggested in the Coleman Q-ball model [27]. The self-interacting Higgs of a Q-ball is confined inside the ball-like source,  $r < R$ , leaving the external vacuum unbroken. Most of the Q-ball models led to a coherent oscillating state of the Higgs vacuum inside the bag<sup>5</sup>(oscillons [29, 30, 31]). The KN soliton-source [13, 14] exhibits also this peculiarity. We can summarize that *confinement of the Higgs condensate inside the bag is necessary requirement for the correct gravitating properties of the bag models*. However, formation of the corresponding potential turns out to be a very non-trivial problem, which cannot be solved by the usual quartic potential (11).

## 4 Field model of broken symmetry and phase transition for gravitating bag model

Among theories with spontaneous symmetry breaking, important place takes the field model of a vortice in condensed matter which was considered by Abrikosov in connection with theory of type II superconductors. Nielsen and Olesen (NO) used this solution for a semiclassical relativistic string model [25]. The NO string model, representing a magnetic flux tube in superconductor, was generalized to many other semiclassical field models of the solitonic strings and found wide application in the electroweak sector of the standard Glashow-Salam-Weinberg (GSW) model [26, 33].

The NO model [25] contains a complex scalar field  $\Phi$  and the gauge EM field  $A^\mu$  which becomes massive through the Higgs mechanism. The La-

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<sup>5</sup>Such a model was first considered by G. Rosen, [28].

grangian has the form

$$\mathcal{L}_{NO} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\mathcal{D}_\mu\Phi)(\mathcal{D}^\mu\Phi)^* - V(|\Phi|), \quad (13)$$

where  $\mathcal{D}_\mu = \nabla_\mu + ieA_\mu$  are the  $U(1)$  covariant derivatives, and  $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$  is the field strength. The potential  $V$  has the same quartic form as in (11)

$$V = \lambda(\Phi^\dagger\Phi - f^2)^2, \quad (14)$$

where  $\sigma$  is replaced by complex field  $\Phi = |\Phi|e^{i\chi}$ .

The Lagrangian  $\mathcal{L}_{NO} \equiv \mathcal{L}_{mat}$  describes a vortex string embedded in the superconducting Higgs condensate *in flat space-time*. Similarly to the bag models, this model cannot be generalized to gravity, since the Higgs condensate gives mass to the external EM field, turning it into a non-physical short-range field conflicting with the external KN solution.

Improvement of this flaw was suggested by Witten in his  $U(1) \times \tilde{U}(1)$  field model of a cosmic superconducting string [24], in which he used two Higgs-like fields,  $\Phi^1$  and  $\Phi^2$ . One of them, say  $\Phi^1$ , had the required behavior, being concentrated inside the source, while another one,  $\Phi^2$ , played auxiliary role and took the external complementary domain extending up to infinity. These two Higgs field are charged and adjoined to two different gauge fields  $A^1$  and  $A^2$ , so that when one of them is long-distant in some region  $\Omega$ , while the second one is long-distant in complimentary region  $\bar{\Omega} = U_\infty \setminus \Omega$ . This model is suitable for any localized gravitating source, however, for superconducting source of the KN solution we used in [13] a supersymmetric generalization of the Witten model suggested by Morris [34].

## 4.1 Supersymmetric phase transition

The supersymmetric scheme of phase transition is based on three chiral fields  $\Phi^{(i)}$ ,  $i = 1, 2, 3$ , [19]. One of this fields, say  $\Phi^{(1)}$ , has the required radial dependence, and we chose it as the Higgs field  $\mathcal{H}$ , setting the additional notations in accord with  $(\mathcal{H}, Z, \Sigma) \equiv (\Phi^0, \Phi^1, \Phi^2)$ .

The coupled with gravity action should reads

$$S = \int \sqrt{-g}d^4x \left( \frac{R}{16\pi G} + \mathcal{L}^{mat} \right), \quad (15)$$

where the full matter Lagrangian takes the form

$$\mathcal{L}^{mat} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\sum_i (\mathcal{D}_\mu^{(i)}\Phi^{(i)})(\mathcal{D}^{(i)\mu}\Phi^{(i)})^* - V, \quad (16)$$

which contains contribution from triplet of the chiral field  $\Phi^{(i)}$ .

The required for our model potential  $V$  is obtained by a standard supersymmetric scheme of broken symmetry [19], which determines it via superpotential  $W(\Phi^i, \bar{\Phi}^i)$ ,

$$V(r) = \sum_i |\partial_i W|^2. \quad (17)$$

The superpotential leading to the required geometry of broken symmetry was suggested by J.Morris [34]:

$$W(\Phi^i, \bar{\Phi}^i) = Z(\Sigma\bar{\Sigma} - \eta^2) + (Z + \mu)\mathcal{H}\bar{\mathcal{H}}, \quad (18)$$

where  $\mu, \eta$  are real constants. It yields

$$V = (Z + \mu)^2|\mathcal{H}|^2 + (Z)^2|\Sigma|^2 + (\Sigma\bar{\Sigma} + \mathcal{H}\bar{\mathcal{H}} - \eta^2)^2, \quad (19)$$

and equation

$$\partial_i W = 0 \quad (20)$$

determines two vacuum states separated by a spike of the potential  $V$  at  $r \approx R$ :

**EXT:** external vacuum,  $r > R + \delta$ ,  $V(r) = 0$ , with vanishing Higgs field  $\mathcal{H} = 0$ , while  $Z = 0, \Sigma = \eta$ , and

**INT:** internal vacuum state,  $r < R - \delta$ ,  $V(r) = 0$ , while  $|\mathcal{H}| = \eta$ ,  $Z = -\mu$ ,  $\Sigma = 0$ ).<sup>6</sup>

## 4.2 Application to KN source

The choose of Lópes's boundary for regularization of the KN source allows us to neglect gravity inside the source and at the boundary, and thus, *one*

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<sup>6</sup>This state could be called a false-vacuum, since vev of the Higgs field is non-zero. However, the term false-vacuum has already been used in the literature in a different sense, and we use here the term "Higgs condensate". Howeve, the term false-vacuum was used in our previous paper [16].

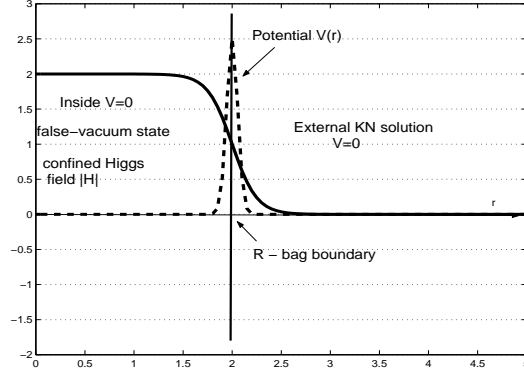


Figure 5: Region of broken symmetry in the KN soliton bag model. The potential  $V(R)$  forms the inner and outer vacuum states  $V = 0$  with a narrow spike at the boundary of the bag. The Higgs field  $H$  is confined inside the bag,  $r < R$ , forming a supersymmetric condensate which gives a mass to the Dirac equation.

can neglect gravitational field in the zone of phase transition and consider the space-time as flat. In the same time, outside the source we have exact Einstein-Maxwell gravity, since the gauge symmetry is unbroken and all the terms  $\frac{1}{2}(\mathcal{D}_\mu \Phi)(\mathcal{D}^\mu \Phi)^*$  vanish together with the potential  $V(|\Phi|)$ . Therefore, outside the source we have only the matter term  $\mathcal{L}^{mat} \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$  leading to the external KN solution.

Therefore, inside the source (zone **INT**) and on the boundary, we have only the part of Lagrangian which corresponds to self-interaction of the complex Higgs field and its interaction with vector-potential of the KN electromagnetic field  $A^\mu$  in the flat space-time.

The field model is reduced to the model, considered by Nielsen-Olesen for a vortex string in the superconducting media [25],

$$\mathcal{L}_{NO} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\mathcal{D}_\mu \mathcal{H})(\mathcal{D}^\mu \mathcal{H})^* - V(|\mathcal{H}|), \quad (21)$$

where  $\mathcal{D}_\mu = \nabla_\mu + ieA_\mu$  is a covariant derivative,  $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ , and  $\nabla_\mu \equiv \partial_\mu$  is reduced to derivative in flat space with a flat D'Alembertian  $\partial_\nu \partial^\nu = \square$ . For interaction of the complex Higgs field

$$\mathcal{H}(x) = |\mathcal{H}(x)|e^{i\chi(x)} \quad (22)$$

with the Maxwell field we obtain the following complicated systems of the nonlinear differential equations

$$D_\nu D^\nu \mathcal{H} = \partial_{\tilde{H}} V, \quad (23)$$

$$\square A_\mu = I_\mu = e|\mathcal{H}|^2(\chi_{,\mu} + eA_\mu). \quad (24)$$

The obtained vacuum states **EXT** and **INT** show that  $|\mathcal{H}(r)|$  should be a step-like function

$$|\mathcal{H}(r)| = \begin{cases} \eta & \text{if } r \leq R - \delta, \\ 0 & \text{if } r \geq R + \delta. \end{cases} \quad (25)$$

with a transition region  $R - \delta < r < R + \delta$  where its behavior is determined by the impact of electromagnetic field.

Outside the source,  $r > R + \delta$ , we have  $\mathcal{H} = 0$  and obtain  $I_\mu = 0$ . Inside the source, by  $r \leq R - \delta$ , we have also  $I_\mu = 0$ , which is provided there by compensation of the vector potential by a gradient of the phase  $\chi$  of the Higgs field,  $\chi_{,\mu} + eA_\mu = 0$ . So, the nonzero current exists only in the narrow transitional region  $R - \delta < r < R$ , where this compensation is only partial, and (24) describes a “region of penetration” of the EM field inside the Higgs condensate, see Fig.5.

### 4.3 Important consequences

Analysis of the equation (24) in [13, 14] showed two remarkable properties of the KN rotating soliton:

(I) The vortex of the KN vector potential  $A_\mu$  forms a quantum Wilson loop placed along the border of the disk-like source, which leads to *quantization of the angular momentum* of the soliton,

(II) the Higgs condensate should *oscillate* inside the source with the frequency  $\omega = 2m$ .

The KN vector potential has the form [21]

$$A_\mu dx^\mu = -Re \left[ \left( \frac{e}{r + ia \cos \theta} \right) (dr - dt - a \sin^2 \theta d\phi) \right]. \quad (26)$$

Maximum of the potential is reached in the equatorial plane,  $\cos \theta = 0$ , at the López’s boundary of the disk-like source (9),  $r_e = e^2/2m$ , which plays the role of a cut-off parameter,

$$A_\mu^{max} dx^\mu = -\frac{e}{r_e} (dr - dt - ad\phi). \quad (27)$$

The  $\phi$ - component of vector potential,  $A_\phi^{max} = ea/r_e$ , shows that the potential forms near the source boundary a circular flow (Wilson loop). According to (24), this flow is compensated inside the soliton by gradient of the Higgs phase  $\chi_{,\phi}$ , and does not penetrate inside the source beyond a transition region  $r < r_e - \delta$ . Integration of this relation over the closed loop  $\phi = [0, 2\pi]$  under condition  $I_\phi = 0$  yields the result **(I)**.

Similarly, using (24) and condition  $I_\phi = 0$  for the time component of the vector potential  $A_0^{max} = \frac{e}{2r_e} = m/e$ , we obtain the result **(II)**.

## 5 Fermionic sector of the KN bag model

Now we have to consider matching of the solutions of Dirac equation with interior of the regular solitonic source and with external KN solution. We start from the region inside the KN source and the adjoined  $\delta$ -narrow layer of phase transition,  $r < R + \delta$ . In accord with the used scheme of regularization, these regions are to be flat, and one can use here the usual Dirac equation  $\gamma^\mu \partial_\mu \Psi = m\Psi$ , which in the Weyl representation splits into two equations

$$\sigma_{\alpha\dot{\alpha}}^\mu i\partial_\mu \bar{\chi}^{\dot{\alpha}} = m\phi_\alpha, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} i\partial_\mu \phi_\alpha = m\bar{\chi}^{\dot{\alpha}}, \quad (28)$$

where the Dirac bispinor  $\Psi = \begin{pmatrix} \phi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$  is presented by two Weyl spinors  $\phi_\alpha$  and  $\bar{\chi}^{\dot{\alpha}}$ .

In the conception of the bag models, fermions acquire mass via a Yukawa coupling to the Higgs field, (12), and since the Higgs condensate in the KN source is concentrated inside the bag, (25), the mass term of the Dirac equation takes the maximal value

$$m = g\eta \quad (29)$$

in internal region, while outside the bag the Dirac equation turns out to be massless, and splits into two independent massless equations

$$\sigma_{\alpha\dot{\alpha}}^\mu i\partial_\mu \bar{\chi}^{\dot{\alpha}} = 0, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} i\partial_\mu \phi_\alpha = 0, \quad (30)$$

corresponding to the left-handed and right-handed “electron-type leptons” of the Glashow-Salam-Weinberg model [20].

Outside the bag we have an external gravitational and EM fields of the KN solution, and one should use the Dirac equation in covariant form

$$\gamma_{KS}^\mu \mathcal{D}_\mu \Psi = 0, \quad (31)$$

where  $\gamma_{KS}^\mu$  are  $\gamma$ -matrixes adapted to the Kerr-Schild form of metric (2), and

$$\mathcal{D}_\mu = \partial_\mu - \frac{1}{2}\Gamma_{\nu\lambda\mu}\Sigma^{\nu\lambda} - i\frac{k}{2\sqrt{2}}\gamma_\mu F_{\nu\lambda}\Sigma^{\nu\lambda} \quad (32)$$

are covariant derivatives.

The exact solutions on KS background were earlier considered by S. Einstein and R. Finkelstein in [35], and following them we can choose the  $\gamma_{KS}^\mu$  matrixes in the form

$$\gamma_{KS}^\mu = \gamma_W^\mu + \sqrt{2H}k^\mu\gamma_W^5, \quad (33)$$

where  $\gamma_W^\mu$  are matrixes of the Weyl representation for Minkowski space  $\eta^{\mu\nu}$ . They satisfy the usual anticommuting relations

$$\{\gamma_W^\mu, \gamma_W^\nu\} = 2\eta^{\mu\nu}, \quad \{\gamma_W^\mu, \gamma_W^5\} = 0, \quad (\gamma_W^5)^2 = -1, \quad (34)$$

while  $\gamma_{KS}^\mu$  satisfy the anticommuting relations

$$\frac{1}{2}\{\gamma_{KS}^\mu, \gamma_{KS}^\nu\} = \eta^{\mu\nu} - 2Hk^\mu k^\nu = g_{KS}^{\mu\nu}, \quad (35)$$

adapted to KS metric. It is known that the exact KS solutions belong to the class of the algebraically special solutions, for which all the tensor quantities are to be aligned with Kerr null congruence, [21], and the general relations (31), (33), (32) become much simpler when the Dirac field  $\Psi(x)$  is “aligned” with the Kerr congruence  $k^\mu(x)$ ,

$$k_\mu\gamma^\mu\Psi = 0. \quad (36)$$

For the aligned Dirac field, the nonlinear terms of the electromagnetic and gravitational interactions are cancelled, and the Dirac equation linearized [35], taking the form of a free Dirac equation in flat space-time (30).

The alignment condition (36) may be rewritten in the form

$$(\vec{k} \cdot \vec{\sigma})\phi = \phi, \quad (\vec{k} \cdot \vec{\sigma})\bar{\chi} = -\bar{\chi}, \quad (37)$$

which shows that the left-handed and the right-handed fields  $\bar{\chi}$  and  $\phi$  are to be oppositely polarized with respect to space direction of the Kerr congruence  $\vec{k}$ . We obtain that only one of these two “half-leptons”, the left-handed  $\phi$  is really consistent with the Kerr congruence  $k^+ = (1, \vec{k})$ , selected for the *physical sheet* of the KN solution. The consistent solution takes the form



$\Psi_L^T = (\phi, 0)$ , which shows explicitly that only left-handed field  $\phi$  is aligned with  $k^+$  and survives on the physical sheet of the KN geometry. This solution is exact, since for the massless Dirac equation the left- and right-handed spinors are independent. Similarly, we obtain the solution  $\Psi_R^T = (0, \bar{\chi})$ , which is not aligned with  $k^+$  and with the selected physical sheet of the KN solution. However, it is aligned with congruence  $k^-$  and can “live” on the negative sheet of advanced fields. Thus, the massive Dirac solution  $\Psi = \begin{pmatrix} \phi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$  splits into the left and right massless parts  $\Psi_L$  and  $\Psi_R$ , which *outside the bag* can live only on the different sheets of the twosheeted Kerr geometry.

This important peculiarity of the Dirac solutions on the Kerr background was also mentioned in [35], where authors noted that the Dirac equations on the KS background “...are not consistent unless the mass vanishes...”. Meanwhile, this obstacle disappears inside the bag-like source of the Kerr geometry, where the space is flat by construction of the solitonic source (sec.2). When the massless Weyl spinors pass from two different external sheets on a common flat space inside the bag, they are combined into a Dirac bispinor which gets mass from the Higgs condensate through Yukawa coupling (see Fig.6). Removing the two-sheeted structure, which was associated with the problem the source of KN solution, we meet its appearance from another side, by analysis of the consistent solutions of the Dirac equation on the KS background. We obtain that two-sheeted structure of the KS geometry is agreed with elementary constituents of the standard model – the massless ‘left-handed’ and ‘right-handed’ electron fields [20, 32], providing consistency of the external Dirac field with KN gravity.

The Kerr congruences are determined by *the Kerr theorem*, [21, 46], which is formulated in twistor term on the auxiliary to KS metric (2) Minkowski space  $\eta_{\mu\nu}$ . The first twistor component,  $Y$  plays also the role of a projective spinor coordinate (see details in Appendix and [16, 46]). The Kerr theorem gives for the KN particle two solutions  $Y^\pm(x)$  which are connected by antipodal relation  $Y^+ = -1/\bar{Y}^-$  and determine two antipodal congruences  $k_{\mu\nu}^+(x)$  and  $k_{\mu\nu}^-(x)$ . The Weyl spinors corresponding to solutions  $Y^\pm(x)$  are exactly the considered above Weyl spinor components  $\phi$  and  $\bar{\psi}$  of the aligned Dirac solutions. Since the Kerr theorem is formulated in the flat space-time, the solutions  $Y^\pm(x)$  are extended unambiguously from the external KN space to

the flat space inside the bag, which determines the Dirac bispinor

$$\tilde{\Psi} = \begin{pmatrix} f_1(x)\phi_\alpha \\ f_2(x)\bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \quad (38)$$

which is aligned to the both external congruences and represents a constraint, selecting in the flat space inside the bag the Dirac solution with required polarization.

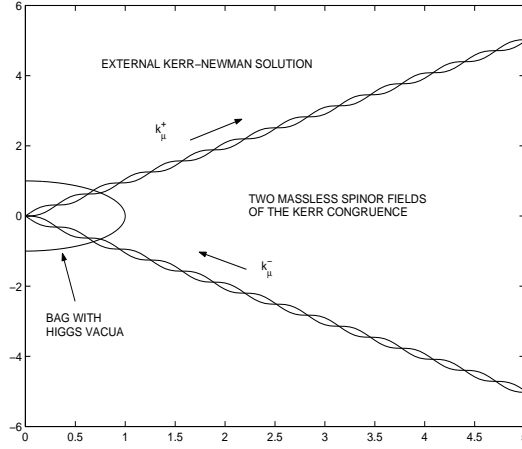


Figure 6: Two sheets of external KN solution are matched with flat space inside the bag. The massless spinor fields  $\phi_\alpha$  and  $\bar{\chi}^{\dot{\alpha}}$  live on different KN sheets, aligned with  $k_\mu^+$  and  $k_\mu^-$  null directions. Inside the bag they join into a Dirac bispinor, which obtains the mass from the Higgs condensate concluded inside the bag.

Another very specific peculiarity of the bag models is emergence of the variable mass term in the Dirac equation (12). The mass term is determined by the vev of Higgs condensate  $\sigma$  which depends on the regions of space-time, and in the region of the maximum of the Higgs condensate  $\sigma = \eta$ , is called the bare mass  $m = g\eta$ . The Dirac wave function, solution of the Dirac equation with variable mass term, avoids the region with a large bare mass, and tends to get a more energetically favorable position, which is principal idea of the quark confinement.

In the SLAC bag model [5], the resulting wave function is determined by variational approach. The Hamiltonian is

$$H(x) = \Psi^\dagger \left( \frac{1}{i} \vec{\alpha} \cdot \vec{\nabla} + g\beta\sigma \right) \Psi, \quad (39)$$

and the energetically favorable wave function is determined by minimization of the averaged Hamiltonian  $\mathcal{H} = \int d^3x H(x)$  under the normalization condition  $\int d^3x \Psi^\dagger(x) \Psi(x) = 1$ . It yields

$$\left(\frac{1}{i}\vec{\alpha} \cdot \vec{\nabla} + g\beta\sigma\right)\Psi = \mathcal{E}\Psi, \quad (40)$$

where  $\mathcal{E}$  appears as the Lagrangian multiplier enforcing the normalization condition. Similar to results of the SLAC-bag model, one expects that the Dirac wave function will not penetrate deeply in the region of large bare mass  $m = g\eta$ , and will concentrate in a very narrow transition zone at the bag border  $R - \delta < r < R + \delta$ . As it was motivated in [5], the narrow concentration of the Dirac wave function is admissible in the bag models, since for the scalar potential there is no the Klein paradox. The exact solutions of this kind are known only for two-dimensional case, and the corresponding variational problem should apparently be solved numerically by using the ansatz (38), where  $f_1(x)$  and  $f_2(x)$  are variable factors.

The use of classical solutions of the Dirac equation in a given scalar potential leads also to the problem of the negative-energy states. In the bag models this problem is considered semiclassically by using the assumption [5] that “...all the negative-energy states in the presence of this potential are filled...”, and as a result, it is necessary to consider only the lowest positive-energy eigenvalues.<sup>7</sup>

Splitting of the Kerr-Schild space-time *outside the source* of KN solution looks strange from the point of view of the standard gravitation, but it appears more natural by comparison with electromagnetism, which is sensitive to difference between the retarded and advanced fields.

It is known, [36], that the Kerr solution may be represented in the Kerr-Schild (KS) form via the both Kerr congruences  $k_\mu^+$  or  $k_\mu^-$ , but not via the both simultaneously. For the KN solution with EM field, situation is more complicated. Although the both representations are admissible, the representation via retarded fields is physically preferable, since the asymptotic advanced EM field of the KN solution will contradict to its experimental

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<sup>7</sup>This is an approximation to rigorous treatment based on the normal-ordering. The negative-energy states correspond to the charge-conjugate solutions  $\Psi^c(x) = C\bar{\Psi}(x)$ ,  $\bar{\Psi}^c(x) = C^{-1}\bar{\Psi}(x)$  of the charge-conjugate Dirac equations (with replacement  $e \rightarrow -e$ ). In particular, the density of current is determined by the commutation relations  $j_\mu(x) = \frac{ie}{2}[\bar{\Psi}(x), \gamma_\mu \Psi(x)] = \frac{ie}{2}(\bar{\Psi}(x)\gamma_\mu \Psi(x) - \frac{ie}{2}\bar{\Psi}^c(x)\gamma_\mu \Psi^c(x))$ , and similar for the expectation value of energy or any other operator bilinear in fermion field.

behavior in flat space. Vector potential  $A_\mu$  of the KN solution must also be aligned with the Kerr congruence, and should be retarded,  $A_{ret}$ , on the physical sheet determined by the outgoing Kerr congruence  $k_\mu^+$ . The appearance of advanced EM fields,  $A_{adv}$  is important in the non-stationary problems. In particular, in the Dirac theory of radiation reaction, the retarded potentials  $A_{ret}$  are split into a half-sum and half-difference with advanced ones  $A_{ret} = \frac{1}{2}[A_{ret} + A_{adv}] + \frac{1}{2}[A_{ret} - A_{adv}]$ , where

$$A_{ret}^+ = \frac{1}{2}[A_{ret} + A_{adv}] \quad (41)$$

is connected with radiation reaction, and

$$A_{ret}^- = \frac{1}{2}[A_{ret} - A_{adv}] \quad (42)$$

forms a self-interaction of the source. Similar structure presents also in the Feynman propagator.

The fields  $A_{ret}$  and  $A_{adv}$  cannot reside on the same physical sheet of the Kerr geometry, because each of them should be aligned with the corresponding Kerr congruence. Considering the retarded sheet as a basic physical sheet, we fix the congruence  $k_\mu^+$  and the corresponding metric  $g_{\mu\nu}^+$ , which are not allowed for the advanced field  $A_{adv}$  and must be positioned on the separate sheet which different metric  $g_{\mu\nu}^-$ .

## 6 Discussion.

Taking the bag model conception, we should also accept their dynamical properties that they soft and easily deformed [5, 17], forming a stringy structure. Typically, these are radial and rotational excitations accompanied by the formation of the open tube-shaped string ended with quarks. Another type of deformations was considered in the Dirac model of an "extensible" electron (1962) [37], which can also be regarded as a prototype bag model with radial excitations.<sup>8</sup> The bag-like source of the KN solution without rotation,  $a = 0$ , coincides with this "extensible" model of the Dirac electron leading to the "classical electron radius"  $R = r_e = e^2/2m$ . As we discussed in Introduction, the disk-like bag of the rotating KN source can be viewed

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<sup>8</sup>This view was also suggested in [38]. Interpretation of the black holes and AdS geometries as a sort of the bag was also noticed in [39, 40].

as stretching of the spherical bag by rotations. For parameters of an electron, the spinning bag stretched in a disk of radius,  $a = \hbar/2mc$ , covering the Compton area of “dressed” electron. The disk is very thin with degree of flattening  $\alpha = 137^{-1}$ . The boundary of disk appears to be very close to former position of Kerr singular ring, and the EM field near the boundary may be seen as a regularization of the KN singular EM field. Similar to other singular lines, the Kerr singular ring was considered as a string in [11]. The structure of the EM field near this string was analyzed first in [10, 11], and much later in [23]. It appeared to be similar to the structure of the fundamental string solution, obtained by Sen to low-energy heterotic string theory [23]. It is a typical light-like pp-wave string solution [43, 44], which in the Kerr geometry takes the ring-like form.

Regularization of the KN source does not remove this ring-string, but gives it a cut-off parameter (9),  $R = r_e$ . It was shown in [10, 11] and later specified in [44, 45, 46] that the EM excitations of the KN solution lead to appearance of traveling waves propagating along this ring-string. However, the light-like ring-string cannot be closed [47], since the points different by angular period,  $x^\mu(\phi, t)$  and  $x^\mu(\phi + 2\pi, t)$  should not coincide, and a peculiar point on the ring-string should make it open, forming a single quark-like endpoint.

The string traveling waves deform the bag boundary creating singular pole [41]. We will not discuss it here in details, leaving the treatment to a separate paper. Note only, that the exact solutions for the EM excitations on the Kerr background were obtained in [21], and using the considered in introduction conditions **I**, and **II** we can unambiguously determine the back reaction of the local EM field on the metric, and obtain the corresponding deformations of the bag boundary. The origin of singular pole is caused by a circulating node in the EM string excitation. This node yields the zero cut off parameter  $R$ , creating contact of the bag boundary with singular ring.

This singular pole circulates along the sharp border of the disk with the speed of the light and may be considered three-fold: a) as a light-like quark enclosed inside the bag, b) as a single end-point of the light-like ring-string (as it was showed in [47], the light-like fundamental string cannot be closed), and c) as a naked point-like electron enclosed in a circular ‘zitterbewegung’. It leads to an integrated model for the dressed and bare electron as a single coherent system similar to the hadronic bag models.

## 7 Conclusion

Starting from the old problem of the source of the KN geometry we obtained first a bubble-core model of the spinning particle, the supersymmetric vacuum state of which is formed by the Higgs mechanism of symmetry breaking. Contrary to the most other known models of the particle-like objects, the KN bubble forms a *gravitating* soliton creating the external gravitational and EM field of an electron. This compatibility with gravity has required the use of a supersymmetric field model of phase transition leading to a supersymmetric vacuum state in the core of the particle, leaving unbroken the external EM field.

The resulting soliton model has much in common with the famous MIT and SLAC bag models, but gets the “dual bag geometry”, in which the Higgs condensate is embedded “inside out ” compared to previous bag models.

The two-sheeted structure of the Kerr geometry has got in this model a natural space-time (coordinate) implementation forming a background for the initially massless leptons of the Glashow-Salam-Weinberg model [20].

Without pretension on the detailed description, we can note that the described dressed electron may be turned in a positron, if we change the role of the advanced and retarded sheets of the Kerr geometry. The higher excitation of the ring-string may generate the muon state, while switching off the scalar and longitudinal components of the EM field, corresponding to charge of the KN solution [48], and preserving only the transversal traveling waves, gives us a neutral particle, which has the features of a neutrino. Therefore, some variations the KN bag model can give us the space-time structure for some other spinning particles of the electroweak sector of the Standard Model.

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## Appendix. The Kerr theorem

The Kerr theorem determines all the geodesic and *shear free* congruences as analytical solutions of the equation

$$F(T^A) = 0, \quad (43)$$

where  $F$  is an arbitrary holomorphic function of the projective twistor variables

$$T^A = \{Y, \zeta - Yv, u + Y\bar{\zeta}\}, \quad A = 1, 2, 3, \quad (44)$$

where  $\zeta = (x + iy)/\sqrt{2}$ ,  $\bar{\zeta} = (x - iy)/\sqrt{2}$ ,  $u = (z + t)/\sqrt{2}$ ,  $v = (z - t)/\sqrt{2}$  are null Cartesian coordinates of the auxiliary Minkowski space.

We notice, that the first twistor coordinate  $Y$  is also a projective spinor coordinate

$$Y = \phi_1/\phi_0, \quad (45)$$

and it is equivalent to two-component Weyl spinor  $\phi_\alpha$ , which defines the null direction<sup>9</sup>  $k_\mu = \bar{\phi}_{\dot{\alpha}}\sigma_\mu^{\dot{\alpha}\alpha}\phi_\alpha$ .

It is known, [21, 46], that function  $F$  for the Kerr and KN solutions may be represented in the quadratic in  $Y$  form,

$$F(Y, x^\mu) = A(x^\mu)Y^2 + B(x^\mu)Y + C(x^\mu). \quad (46)$$

In this case (43) can explicitly be solved, leading to two solutions

$$Y^\pm(x^\mu) = (-B \mp \tilde{r})/2A, \quad (47)$$

where  $\tilde{r} = (B^2 - 4AC)^{1/2}$ . It has been shown in [46], that these solutions are antipodally conjugate,

$$Y^+ = -1/\bar{Y}^-. \quad (48)$$

Therefore, the solutions (47) determine two Weyl spinor fields  $\phi_\alpha$  and  $\bar{\chi}_{\dot{\alpha}}$ , which in agreement with (48) are related with two antipodal congruences

$$Y^+ = \phi_1/\phi_0, \quad (49)$$

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<sup>9</sup>We use the spinor notations of the book [19], where the  $\sigma$ -matrixes has the form  $\sigma^\mu = (1, \sigma^i)$ ,  $\bar{\sigma}^\mu = (1, -\sigma^i)$ ,  $i = 1, 2, 3$  and  $\sigma^\mu = \sigma_{\alpha\dot{\alpha}}^\mu$ ,  $\bar{\sigma}^\mu = \bar{\sigma}^{\mu\dot{\alpha}\alpha}$ .

$$Y^- = \bar{\chi}_1 / \bar{\chi}_0. \quad (50)$$

In the Debney-Kerr-Schild (DKS) formalism [21] function  $Y$  is also a *projective angular coordinate*  $Y^+ = e^{i\phi} \tan \frac{\theta}{2}$ . It gives to spinor fields  $\phi_\alpha$  and  $\bar{\chi}_{\dot{\alpha}}$  an explicit dependence on the Kerr angular coordinates  $\phi$  and  $\theta$ .

For the congruence  $Y^+$  this dependence takes the form

$$\phi_\alpha = \begin{pmatrix} e^{i\phi/2} \sin \frac{\theta}{2} \\ e^{-i\phi/2} \cos \frac{\theta}{2} \end{pmatrix}. \quad (51)$$

In agreement with (48) we have  $\bar{Y}^- = -e^{-i\phi} \cot \frac{\theta}{2}$ , and from invariant normalization  $\phi_\alpha \chi^\alpha = 1$  we obtain  $\chi_\alpha = \begin{pmatrix} -e^{i\phi/2} \cos \frac{\theta}{2} \\ e^{-i\phi/2} \sin \frac{\theta}{2} \end{pmatrix}$  which yields

$$\bar{\chi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\chi}_{\dot{\beta}} = \begin{pmatrix} e^{i\phi/2} \sin \frac{\theta}{2} \\ e^{-i\phi/2} \cos \frac{\theta}{2} \end{pmatrix}. \quad (52)$$

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